Load Flow Model Specification for the Calculation of Nodal Transmission Loss Factors

~~Version 2.0~~

Effective Date: ~~12 May 2017~~

AMENDMENT HISTORY

Implementation Date	Version	Description of Change	Mods/ Panel/ Committee Refs
June 2003	1.0	Baseline version for P82	CVA Programme
12 May 2017	2.0	Modification P350	Panel 266/10
29/03/2019	2.2	29 March 2019 Standalone Release for P369	P369

CONTENTS

1 INTRODUCTION

[P369]This Load Flow Model (LFM) Specification has been established by the Panel in accordance with Balancing and Settlement Code (BSC) Section T Annex T-2 paragraphs 2 and 3, with support from BSCCo and the ~~Transmission Company~~National Electricity Transmission System Operator (NETSO). This LFM Specification is a Code Subsidiary Document, which forms part of the Service Description of the Transmission Loss Factor Agent (TLFA) (reference 1).

The LFM Specification contains the requirements, obligations, assumptions and approximations required to be supported by the LFM. The exact mechanism for the derivation of Nodal Transmission Loss Factors (TLFs) by the TLFA is the required function of the LFM. For the avoidance of doubt, the LFM produces Nodal TLFs and any further data manipulation is carried out by the Transmission Loss Factor Agent. For example, converting the Nodal TLFs into Zonal TLFs and then into BM Unit specific TLFs.

In the event of any discrepancy between the LFM Specification and the TLFA Service Description, Section H 1.5.2 (b) of the Code places the obligation on the Panel, with support from BSCCo, to determine the precedence and resolve any discrepancy by raising the relevant amendment in accordance with Section F.3. of the Code. Furthermore, in the event of any discrepancy between the LFM Specification and the Code Section H 1.5.2 (b) places the obligation on the Panel, with support from BSCCo, to determine precedence, and to resolve the discrepancy by raising the relevant amendment, in accordance with the Code, Section F 3.

1.1 Model Reviewer

The BSC Panel will appoint a Model Reviewer who will verify that the LFM produced by the TLFA produces Nodal TLFs in accordance with this LFM Specification, in accordance with Section T, Annex T-2 of the Code. Load Flow Model

1.2 Background

The intent of Load Flow Model is to derive a set of annual average Transmission Loss Factors (TLFs) to recover heating losses on a zonal basis and fixed losses on a uniform basis, using a scaling factor of 0.5. The TLFs are to be derived annually on an ex ante basis using historical metered and network data. Nodal TLFs will be derived for a set of Sample Settlement Periods.

A Load Flow Model to be used for evaluation of TLFs is to be based on a DC load flow, i.e. a modelling approach for an interconnected network utilising data reflective of alternating current (AC) electrical flows on that network, but with a set of simplifying assumptions that render the equations for the AC flows similar in form to those for a direct current (DC) flow. The Load Flow Model Produces Nodal TLFs for each Sample Settlement Period.

1.3 Objectives

A LFM is a mathematical model of an electrical network which represents power flows between pairs of adjacent nodes on the network, and from which Nodal TLFs can be determined for each Node for given power flows. TLFs are representative of the changes in transmission losses arising from marginal changes in demand or generation at Nodes on the Transmission Network.

The key objectives of the LFM are to:

1. Accurately represent the physical characteristics of the England and Wales Transmission Network via a direct current (DC) load flow model;

2. Use Network Data that reflects, as far as is reasonably possible, the conditions prevailing on the network at any time, representative of an 'intact network', i.e. a complete England and Wales Transmission Network assuming no circuits de-energised or disconnected with all lines in operation;

3. Capture the delivery, injections onto the network, and offtake, withdrawals from the network, for a large number of Nodes for Sample Settlement Periods throughout each Reference Year; and

4. Generate TLFs that are representative of the changes in transmission losses arising from marginal changes in demand or generation at nodes on the Transmission Network.

1.4 Assumptions and Approximations

[P369]The Load Flow Model Specification shall provide for the following assumptions and approximations to be made in the Load Flow Model:

1. Only electrical losses associated with power flows on circuits (forming part of the network) will be used in determining Nodal TLFs (fixed losses will be set in line with those in the ~~Transmission Company~~NETSO Seven Year Statement)

2. In respect of the power flow between adjacent nodes, it is assumed that:

    a) There is no Reactive Power component;

    b) The ratio of the change of power flow over a circuit to the injection at a given node is not dependent on overall electrical load on the network;

    c) The sine of the voltage phase angle is equal to the phase angle (as measured in radians); and

    d) The power flow in a circuit is equal to the difference in the voltage phase angles across the circuit multiplied by the circuit susceptance.

2 LOAD FLOW MODEL REQUIREMENTS

On the basis of the required assumptions listed above, the specification of an appropriate DC load flow model is presented in the following sections (3.1 to 3.3). A conventional DC Load Flow Model relates real power flows (i.e. generation or demand MW) to voltage phase angle (voltage magnitude being assumed constant and equal to 1 pu) using only branch reactances, all resistance being ignored.

The process of computing TLFs based on such a DC Load Flow Model will involve the following three steps:

- STEP 1: Calculate adjusted nodal power flows from Nodal metered generation and demand data, suitable for the application of the conventional DC Load Flow Model;
- STEP 2: Calculate network power flows using the conventional DC Load Flow Model;
- STEP 3: Determine flow-injections sensitivity factors and compute TLFs

These steps are detailed in the following section 3.1 to 3.3 and an example is contained in Appendix 2.

2.1 STEP 1: Calculation of adjusted nodal power flows from metered generation and demand data

The conventional DC Load Flow Model excludes consideration of losses in the process of evaluating voltage phase angles and flows. It is proposed that a simple adjustment of metered volumes of generation (MWh) and demand (MWh) is performed and used to compute Nodal power flows as the input to the DC Load Flow Model:

$$G_n = \overline{G}_n (1 - \frac{L}{2\sum \overline{G}}) \text{ and } \quad (1)$$

$$D_n = \overline{D}_n (1 + \frac{L}{2\sum \overline{D}}) \quad (2)$$

where

$\overline{G}_n$ and $\overline{D}_n$ are metered generation and demand respectively at Node *n*, and where *L* is the metered losses calculated as follows:

$$L = \sum \overline{G}_n - \sum \overline{D}_n \quad (3)$$

G_n and D_n are Nodal power flows to be used in the DC Load Flow Model

This adjustment allows the conventional (loss-inclusive) DC Load Flow Model to be applied for the evaluation of network power flows since:

$$\sum G_n - \sum D_n = 0 \quad (4)$$

Note that this process will produce consistent inputs for the DC Load Flow even if the metered data is inconsistent. For example, in case that the metered losses are inconsistent with metered generation and demand, as well as in the extreme case of the total metered generation being smaller than the total metered demand. The example presented in the Appendix 2 illustrates the adequacy of the proposed approach to computing Nodal power flows from the metered data.

2.2 STEP 2: Evaluation of network power flows using the conventional DC Load Flow Model

Active power balance at each of the Nodes is given by the following expression:

$$G_a - D_a = P_a = \sum_{n=1}^{N} |E_a||E_n|[G_{an}\cos(\theta_a - \theta_n) + B_{an}\sin(\theta_a - \theta_n)] \quad a = 1,..,n \qquad (5)$$

Where

$P_a = G_a - D_a$ The net Nodal power flow, defined as the difference between generation and demand at the corresponding Node

θ_a, θ_n the phase angles at Nodes *a* and *n* respectively

$|E_a|,|E_n|$ the voltage magnitudes at Nodes *a* and *n* respectively

$G_{an} + jB_{an} = Y_{an}$ - the *a-n* term in the complex Y matrix of the power network

The conventional DC load flow is obtained by

(i) neglecting losses in power flow calculations $G_{ab} = 0$,

(ii) assuming that the voltage magnitudes at all Nodes equal to 1 p.u ($|E_n| = 1$).

(iii) assuming that the sine of the voltage phase angle is equal to the phase angle:

$\sin(\theta_a - \theta_n) \approx (\theta_a - \theta_n)$

The corresponding load flow equations constitute a DC power flow:

$$P_a = \sum_{n=1}^{N} B_{an}(\theta_a - \theta_n) \qquad (6)$$

given that

$$B_{ab} = \frac{1}{x_{an}} \qquad (7)$$

where, x_{an} is the reactance between Nodes *a* and *n*, the corresponding conventional DC Load Flow Model can be presented the standard matrix form:

$$\begin{bmatrix} P_1 \\ P_2 \\ \dots \\ P_N \end{bmatrix} = \begin{bmatrix} \sum_{n \neq 1} \frac{1}{x_{1n}} & -\frac{1}{x_{12}} & \dots & -\frac{1}{x_{1N}} \\ -\frac{1}{x_{21}} & \sum_{n \neq 2} \frac{1}{x_{2n}} & \dots & -\frac{1}{x_{2N}} \\ \dots & \dots & \dots & \dots \\ -\frac{1}{x_{n1}} & \dots & \dots & \sum_{n \neq N} \frac{1}{x_{Nn}} \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dots \\ \theta_N \end{bmatrix} \qquad (8)$$

where:

$P_1,..,P_N$ represents net power flow injections (given) at Nodes 1 to *N*,

$\theta_1, \dots, \theta_N$ voltage phase angles (to be calculated) at Nodes 1 to *N*, and

x_{ab} is the reactance of the circuits between Nodes *a* and *b* (given).

The net power flow is defined as the difference between generation and demand at the corresponding Node ($P_n = G_n - D_n$).

The matrix representing network characteristics (both the topology and electrical parameters of the circuits - reactances), belongs to the class of so-called Y_{bus} matrices, and is presented in (9). The diagonal elements of the matrix correspond to the sum of susceptances coincident with the corresponding Node, while off diagonal elements correspond to the negative values of susceptance linking the corresponding Nodes.

$$[Y_{bus}] = \begin{bmatrix} \sum_{n \neq 1} \frac{1}{x_{1n}} & -\frac{1}{x_{12}} & \dots & -\frac{1}{x_{1N}} \\ -\frac{1}{x_{21}} & \sum_{n \neq 2} \frac{1}{x_{2n}} & \dots & -\frac{1}{x_{2N}} \\ \dots & \dots & \dots & \dots \\ -\frac{1}{x_{n1}} & \dots & \dots & \sum_{n \neq N} \frac{1}{x_{Nn}} \end{bmatrix} \qquad (9)$$

In order to solve system of equations (8) a reference slack node needs to be chosen, since (9) is a singular matrix and hence equations (8) are linearly dependent. With no loss of generality but for the sake of simplicity of the presentation, Node 1 is declared as the slack node. The system of equation (8) can be now solved and the corresponding voltage phase angles determined using matrix techniques routinely applied in load flow calculations:

$$[\theta] = \begin{bmatrix} 0 & 0 \\ 0 & [Y_r]^{-1} \end{bmatrix} [P] \qquad (10)$$

where Y_r is obtained by removing the row and the column from the Y_{bus} that correspond to the slack node.

Once the voltage phase angles are calculated (10), circuit flows can be computed:

$$F_k = \frac{1}{x_{ab}}(\theta_a - \theta_b) \qquad (11)$$

where F_k is the power flow in a circuit *k*, and circuit *k* is between Nodes *a* and *b*.

2.3 STEP 3: Determine power injection sensitivity factors and compute TLFs

A Nodal TLF, associated with a particular Node *n*, is defined as the incremental change in the network losses (*L*) due to an incremental increase in power injection (P_n) at Node *n*:

$$TLF_n = \frac{\Delta L}{\Delta P_n} \qquad (12)$$

(Symbol Δ indicates an incremental change)

As indicated above, network losses will be divided in "heating losses", which depend on network loading conditions, and "fixed losses" that are independent from network loading. Therefore, the network model to be used for Nodal TLF evaluations will only include components that generate heating losses, which means that the network model will contain only series impedances and exclude all shunt impedances.

In a network with the total number of circuits (network branches) being *M*, the total "heating losses" are the sum of losses attributed to each individual transmission circuits *k* in the network:

$$L^h = \sum_{k=1}^{M} L_k^h \qquad (13)$$

The Nodal TLF associated with Node n can now be expressed as follows:

$$TLF_n = \frac{\Delta L^h}{\Delta P_n} = \frac{\Delta}{\Delta P_n}\left(\sum_{k=1}^{M} L_k^h\right) = \sum_{k=1}^{M_c} \frac{\Delta L_k^h}{\Delta P_n} \qquad (14)$$

Consistent with the conventional DC Load Flow Model, the heating losses in each of the individual circuits can be assessed as follows:

$$L_k^h = r_k F_k^2 \qquad (15)$$

where:

F_k - per unit active flow through transmission circuit k , and

r_k - per unit resistance of transmission circuit *k*.

Given that F_k is calculated through the conventional DC Load Flow Model, the Nodal TLF for a particular Node n is now given by

$$TLF_n = \sum_{k=1}^{M} \frac{\Delta L_k^h}{\Delta P_n} = \sum_{k=1}^{M} \frac{\Delta (r_k F_k^2)}{\Delta P_n} \qquad (16)$$

This expression can be further expanded as follows:

$$TLF_n = \sum_{k=1}^{M} 2 r_k F_k \frac{\Delta F_k}{\Delta P_n} \qquad (17)$$

The above expression is fundamental for the evaluation of the Nodal TLFs using the required DC Load Flow approach. The sensitivity factor $\frac{\Delta F_k}{\Delta P_n}$ in (17) measures the change in the power flow in circuit k due to an increase in power injection at Node n. In the conventional DC Load Flow Model, these sensitivity factors do not depend on loading conditions but only on the network topology and reactances of the network circuits. Hence, for a network with a fixed topology the sensitivity factors are constant and are evaluated without considering generation and demand.

This is consistent with the requirement set in Section 2.3, point 2(ii). However, the Nodal TLFs (in expression 17) do depend on loading conditions since load flows in individual circuits (F_k) will be driven by loading conditions.

The sensitivity factors, the ratio of the change of power flow F_k, between Nodes a and b, to the increase in power flow P_n at node n can be calculated from the following expression:

(18)

$$h_{kn} = \frac{\Delta F_k}{\Delta P_n} = \frac{\Delta}{\Delta P_n}\left[\frac{1}{x_{ab}}(\theta_a - \theta_b)\right] = \frac{1}{x_{ab}}\left(\frac{\Delta \theta_a}{\Delta P_n} - \frac{\Delta \theta_b}{\Delta P_n}\right)$$

Given that (10) is expressed in the form of

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \cdots \\ \theta_N \end{bmatrix} = \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ 0 & X_{22} & \cdots & X_{2N} \\ 0 & ,,, & \cdots & \cdots \\ 0 & X_{N2} & \cdots & X_{NN} \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \\ \cdots \\ P_N \end{bmatrix} \qquad (19)$$

the sensitivity factors are obtained by the following expression

$$h_{kn} = \frac{\Delta F_k}{\Delta P_n} = \frac{1}{x_{ab}}(X_{an} - X_{bn}) \qquad (20)$$

where:

$$\begin{bmatrix} 0 & 0 \\ 0 & [Y_r]^{-1} \end{bmatrix} = \begin{bmatrix} 0 & \dots & \dots & 0 \\ 0 & X_{22} & \dots & X_{2N} \\ 0 & ,,, & \dots & \dots \\ 0 & X_{N2} & \dots & X_{NN} \end{bmatrix}$$

X_{an} and X_{bn} are the entries of the inverse of the reduce Y_{bus} matrix (19) positioned in rows *a* and *b*, respectively, and in column *n*.

These factors can be readily computed using matrix techniques routinely employed in load flow calculations. The sensitivity factors only depend on values of network parameters but not on network loading.

The values of the sensitivity coefficients depend on the choice of slack node and therefore, the values of Nodal TLFs (17) will also depend on the choice of slack node. However, the differences in TLFs between any two nodes (TLF differentials) will remain constant irrespective of the choice of slack node, since the differences in sensitivity factors are also independent from the choice of slack node.

The above Nodal TLFs, as defined in (17), represent the incremental change in losses due to an incremental increase in power flow, i.e. incremental generation. Given that the formulas used to calculate TLMOs assume demand oriented definition, the polarity of these Nodal TLFs should be reversed for the subsequent application:

$$TLF_n = -\sum_{k=1}^{M} 2 r_k F_k \frac{\Delta F_k}{\Delta P_n} \qquad (21)$$

3 COMPLIANCE

The LFM should be compliant with the LFM Specification (this document) at all times. The LFM should not be adopted, nor amendments implemented until the model reviewer has reported on the compliance of the LFM with the specification and the Panel has agreed that the LFM is compliant with the LFM specification.

The Panel is required to agree to any amendment to the LFM Specification, and therefore the LFM, and is required to instruct the TLFA to amend the LFM to comply with the amendments to the specification.

4 APPENDIX 1 - DEFINITIONS AND TERMS

Adjusted nodal power flows	A form of nodal power flows used to calculate Nodal Transmission Loss Factors in accordance with BSC Section T, Annex T-2.
BSC Year	each successive period of 12 months beginning on 1st April in each year.
Load Periods	Division of the Reference Year into a number of different periods representing typically different levels of load on the Transmission System. Load Periods are mutually exclusive and do not overlap.
Network Data	means the following data relating to the Transmission System: (i) the identity of each pair of adjacent Nodes; (ii) for each such pair of Nodes, value of resistance and reactance between the Nodes; Network data shall be established on the assumption of an 'intact network', that is disregarding any planned or other outage of any part of the Transmission System.
Node	a node is a point on the electrical network at which: (i) a power flow on to or off the network can occur, or (ii) two or more circuit (forming part of the network) meet. A Node refers to nodes on the Transmission System.
Reference Year	12 month period ending 30 September in the preceding BSC Year.
Sample Settlement Period	a representative Settlement Period within a Load Period. Every Sample Settlement Period in the Reference Year falls into one and only one Load Period.
slack node	is a node that acts: (i) in relation to adjacent nodes, as the reference node for calculating the phase angle of the power flow between nodes.
Transmission Loss Factor	is the factor applied to a BM Unit in a Settlement Period in order to adjust for Transmission Losses.
Transmission Loss Factor Agent	the BSC Agent responsible for producing Zonal Transmission Loss Factors and BM Unit specific Transmission Loss Factors.
Zone	a geographic area in which a GSP Group lies, determined by the Panel but so that the Zones are mutually exclusive and comprise of the whole of (and nothing but) the authorised area under the Transmission Licence.

5 APPENDIX 2 - ILLUSTRATIVE EXAMPLE

Consider a simple three-Node network with three circuits in Figure 1, with given metered generation and demand volumes.

Circuit per unit reactances and resistances, assuming 100MVA base, are given in Table 1:

Circuit	Per unit reactance	Per unit resistance
1-2	0.1	0.02
1-3	0.2	0.03885
2-3	0.2	0.04

An advanced (non-standard), loss-inclusive DC Load Flow Model was used to determine individual flows that approximately correspond to the given metered data. This load flow uses a piece wise linear representation of losses in individual circuits. Results of these calculations are shown in Figure 1, with sending and receiving powers in individual lines being presented.

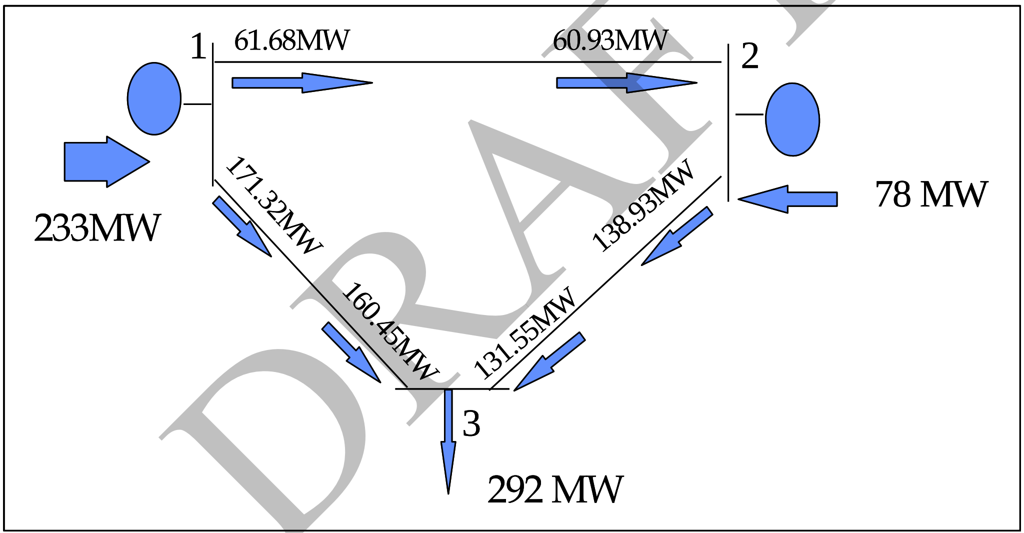

Figure 1: Example system with metered volumes and load flows in individual circuits

Nodal TLFs can be calculated in the process composed of the following steps.

5.1 STEP 1: Adjust metered volumes

From given metered generation and demand data, metered heating losses are found to be 19MW (=233+78-292). Generation and demand are now adjusted to allow the application of the loss-inclusive DC Load Flow Model:

$$G_1 = \overline{G}_1 (1 - \frac{L}{2\sum \overline{G}}) = 233 \cdot (1 - \frac{19}{2 \cdot 311}) = 225.9\,\text{MW}$$

$$G_2 = \overline{G}_2 (1 - \frac{L}{2\sum \overline{G}}) = 78 \cdot (1 - \frac{19}{2 \cdot 311}) = 75.6\,\text{MW}$$

$$D_3 = \overline{D}_3 (1 + \frac{L}{2\sum \overline{D}}) = 292 \cdot (1 + \frac{19}{2 \cdot 292}) = 301.5\,\text{MW}$$

Clearly, the total adjusted generation equals the total adjusted demand.

5.2 STEP 2: Calculate network flows consistent using the conventional DC Load Flow Model

From given set of reactances and the network topology, the following Y_{bus} can be formed:

$$[Y_{bus}] = \begin{bmatrix} 15 & -10 & -5 \\ -10 & 15 & -5 \\ -5 & -5 & 10 \end{bmatrix}$$

By removing the row and the column from the admittance matrix Y_{bus} that corresponds to the slack node, matrix Y_r is obtained. Assuming Node 1 is selected to be a slack, Y_r is defined as:

$$Y_r = \begin{bmatrix} 15 & -5 \\ -5 & 10 \end{bmatrix}$$

Given the reduced matrix Y_r and power injections, voltage phase angles can be computed:

$$\begin{bmatrix} 0 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.08 & 0.04 \\ 0 & 0.04 & 0.12 \end{bmatrix} \cdot \begin{bmatrix} 2.259 \\ 0.756 \\ -3.015 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.06012 \\ -0.3316 \end{bmatrix}$$

Finally, circuit load flows can be computed:

$$F_{1-2} = \frac{1}{x_{1-2}}(\theta_1 - \theta_2) = \frac{1}{0.1}(0-(-0.06012)) = 0.6012\,pu \qquad (60.12\text{MW})$$

$$F_{1-3} = \frac{1}{x_{1-3}}(\theta_1 - \theta_3) = \frac{1}{0.2}(0-(-0.3316)) = 1.658\,pu \qquad (165.8\text{MW})$$

$$F_{2-3} = \frac{1}{x_{2-3}}(\theta_2 - \theta_3) = \frac{1}{0.2}(-0.06012-(-0.3316)) = 1.357\,pu \qquad (135.7\text{MW})$$

This is presented in Figure 2.

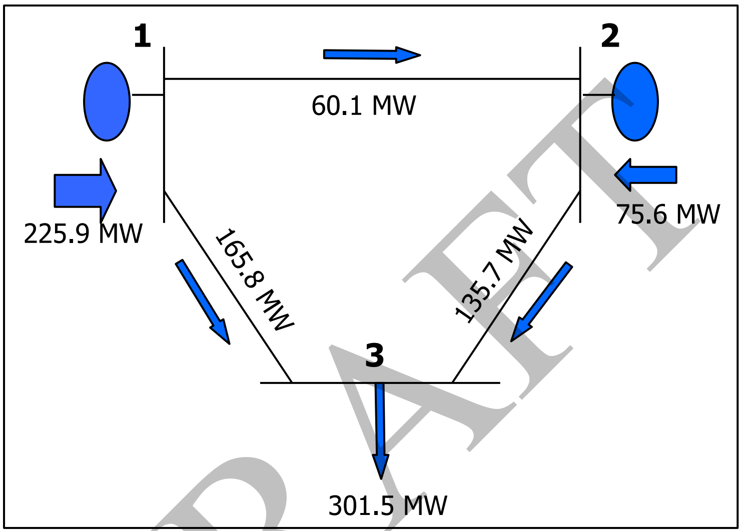

Figure 2: Nodal Power flows in the example system with adjusted generation and demand volumes (total adjusted generation equal total adjusted demand)

5.3 STEP 3: Determine flow-injections sensitivity factors and compute TLFs

Given the entries of the inverse Y_r matrix

$$\begin{bmatrix} 0 & 0 \\ 0 & [Y_r]^{-1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.08 & 0.04 \\ 0 & 0.04 & 0.12 \end{bmatrix},$$

flow-injection sensitivities can be computed:

5.3.1 Circuit 1-2

$$\frac{\Delta F_{1-2}}{\Delta P_1} = \frac{1}{x_{1-2}}(X_{11} - X_{21}) = \frac{1}{0.1}(0-0) = 0$$

$$\frac{\Delta F_{1-2}}{\Delta P_2} = \frac{1}{x_{1-2}}(X_{12} - X_{22}) = \frac{1}{0.1}(0-0.08) = -0.8$$

$$\frac{\Delta F_{1-2}}{\Delta P_3} = \frac{1}{x_{1-2}}(X_{13} - X_{23}) = \frac{1}{0.1}(0-0.04) = -0.4$$

5.3.2 Circuit 1-3

$$\frac{\Delta F_{1-3}}{\Delta P_1} = \frac{1}{x_{1-3}}(X_{11} - X_{31}) = \frac{1}{0.2}(0-0) = 0$$

$$\frac{\Delta F_{1-3}}{\Delta P_2} = \frac{1}{x_{1-3}}(X_{12} - X_{32}) = \frac{1}{0.2}(0-0.04) = -0.2$$

$$\frac{\Delta F_{1-3}}{\Delta P_3} = \frac{1}{x_{1-3}}(X_{13} - X_{33}) = \frac{1}{0.2}(0-0.12) = -0.6$$

5.3.3 Circuit 2-3

$$\frac{\Delta F_{2-3}}{\Delta P_1} = \frac{1}{x_{2-3}}(X_{21} - X_{31}) = \frac{1}{0.2}(0-0) = 0$$

$$\frac{\Delta F_{2-3}}{\Delta P_2} = \frac{1}{x_{2-3}}(X_{22} - X_{32}) = \frac{1}{0.2}(0.08-0.04) = 0.2$$

$$\frac{\Delta F_{2-3}}{\Delta P_3} = \frac{1}{x_{2-3}}(X_{23} - X_{33}) = \frac{1}{0.2}(0.04-0.12) = -0.4$$

Applying equation (x), nodal TLFs can be calculated:

$$TLF_1 = 0$$

$$TLF_2 = 2(r_{1-2}F_{1-2}\frac{\Delta F_{1-2}}{\Delta P_2} + r_{1-3}F_{1-3}\frac{\Delta F_{1-3}}{\Delta P_2} + r_{2-3}F_{2-3}\frac{\Delta F_{2-3}}{\Delta P_2}) =$$

$$= 2(0.02x0.6012x(-0.8) + 0.03885x1.658x(-0.2) + 0.04x1.357x0.2) = -0.0232$$

$$TLF_3 = 2(r_{1-2}F_{1-2}\frac{\Delta F_{1-2}}{\Delta P_3} + r_{1-3}F_{1-3}\frac{\Delta F_{1-3}}{\Delta P_3} + r_{2-3}F_{2-3}\frac{\Delta F_{2-3}}{\Delta P_3}) =$$

$$= 2(0.02x0.6012x(-0.4) + 0.03885x1.658x(-0.6) + 0.04x1.357x(-0.4)) = -0.1303$$

Table 2: Nodal TLFs for incremental increases in generation and demand

Busbar	TLFs for incremental increase in generation	TLFs for incremental increase in demand
1	0.0	0.0
2	-0.0232	0.0232
3	–0.1303	0.1303

The TLFs that represent the incremental change in losses due to incremental change in demand (21) should be used for subsequent calculations of TLMOs.