Optimisation

ESCS

Optimal Calculation of Scaling Weights for GSP Group Correction

Demand Forecasting, EDF Energy

EDF ENERGY

Calculation of Optimal Scaling Weights

Type	Annual Volume (TWh)	Allocation Error	Metered /Losses Error Correlation	01/04/2014 Scaling Weight	Calculated Scaling Weight	Calculated Scaling Weight (Ignoring Correlation)
NHH Metered	171	5.3%	45%	1	1	1
NHH Losses	9	11.9%	45%	2.25	1.211	0.277
HH Metered	129	0.52%	10%	0.1	0.007	0.007
HH Losses	6	5%	10%	0.94	0.034	0.029

* Input data sources:

- Annual Volume (TWh) derived from Table 3 of Aug13 Consultation Document.
- Allocation Error from Table 4 of Aug13 Consultation Document
- Metered/Losses Error Correlation from EDF Energy analysis

Formula for Calculating Optimal Scaling Weights

On the previous slide we have used this formula:

$$GSPGCSW_i \propto (volume_i * std\%_i^2 + correlation_{ik} * std\%_i * std\%_k * volume_k)$$

where:
- subscripts i and k indicate correlated CCC groups (e.g. NHH consumption and NHH loss);
- $std\%_i$ is the percentage standard deviation given in the ELEXON consultation paper;
- $volume_i$ is an average MW level;
- $correlation_{ik}$ is the percentage correlation between the errors before group correction in the related CCC groups i and k.

Overview of Formula Derivation

A – Link BSC Scaling Weights to the standard statistical result for optimal error allocation fraction

B – Present the derivation of the standard statistical result

1. Express the components of the error for each CCC group after group correction;
2. Use this expression to find the variance of the error after group correction in terms of allocation fraction
3. Find the allocation fraction that minimises the variance – ie the allocation of error that maximises the accuracy of group corrected settlements

Derivation A (1)
Linking GSPGCSW with standard statistics

BSC Calculation of GSPGCF:

$$CF = 1 + \frac{V_T - \sum_i^n V_{GC_i}}{\sum_i^n (V_{GC_i} * SW_i)}$$

Where

V_T is the GSP group take,

V_{GC_i} is the consumption in each CCC group within a GSP,

SW_i is the scaling weight for each group.

Derivation A (2)
Linking GSPGCSW with standard statistics

BSC formula for final adjusted volume of CCC group, substituting in definition of correction factor and rearranging:

$$\hat{V}_{GC_i} = V_{GC_i} + V_{GC_i} * \left(SW_i * (CF - 1)\right)$$
$$= V_{GC_i} + V_{GC_i} * SW_i * \left(\frac{V_T - \sum_i^n V_{GC_i}}{\sum_i^n (V_{GC_i} * SW_i)}\right)$$
$$= V_{GC_i} + \left(V_T - \sum_i^n V_{GC_i}\right) * \left(\frac{V_{GC_i} * SW_i}{\sum_i^n (V_{GC_i} * SW_i)}\right)$$

Key points:

$(V_T - \sum_i^n V_{GC_i})$ is the total error

$\left(\frac{V_{GC_i} * SW_i}{\sum_i^n (V_{GC_i} * SW_i)}\right)$ is the error allocation fraction of CCC group i.

Derivation A (3)
Linking GSPGCSW with standard statistics

Compare with the standard formula for statistical error allocation fraction:

$$\frac{\mathrm{std}_\mathrm{i}^2 + \mathrm{correlation}_\mathrm{ik} * \mathrm{std}_\mathrm{i} * \mathrm{std}_\mathrm{k}}{\mathrm{sum}(\mathrm{std}_\mathrm{j}^2) + \ \mathrm{sum}(\mathrm{correlation}_\mathrm{jk} * \mathrm{std}_\mathrm{j} * \mathrm{std}_\mathrm{k})}$$

where std is the standard deviation, and $correlation_{ik}$ indicates the correlation between group i and k.

Set the expressions equal to evaluate Scaling Weight for optimal allocation:

$$\frac{V_{GC_i} * SW_i}{\sum_i^n (V_{GC_i} * SW_i)} = \frac{\mathrm{std}_\mathrm{i}^2 + \mathrm{correlation}_\mathrm{ik} * \mathrm{std}_\mathrm{i} * \mathrm{std}_\mathrm{k}}{\mathrm{sum}(\mathrm{std}_\mathrm{j}^2) + \ \mathrm{sum}(\mathrm{correlation}_\mathrm{jk} * \mathrm{std}_\mathrm{j} * \mathrm{std}_\mathrm{k})}$$

Note: denominator is fixed across all CCCs, and Scaling Weights are " relative" values, therefore we can consider only the numerators:

$$(V_{GC_i} * SW_i) \ \propto (\mathrm{std}_\mathrm{i}^2 + \mathrm{correlation}_\mathrm{ik} * \mathrm{std}_\mathrm{i} * \mathrm{std}_\mathrm{k})$$

Derivation A (4)
Linking GSPGCSW with standard statistics

Typically CCC error levels are quoted in % rather than absolute terms.

Therefore for practical use we expand the statistical result:

$$std_i = volume_i * \ std\%_i$$

where $std\%_i$ is the percentage standard deviation.

Dividing through by volume gives

$$\boxed{SW_i \propto (volume_i * std\%_i^2 + \ correlation_{ik} * std\%_i * std\%_k * volume_k)}$$

Derivation B (1)
Standard statistical error allocation

Statistical expectation

$E[x]$ is the statistical expectation or "average" value of the random variable x. Note the following property:

$$E[x_1 + x_2 + x_3] = E[x_1] + E[x_2] + E[x_3]$$

Variance

If the variable[1] ε_i has a zero mean,

$$\sigma_i^2 = E[\varepsilon_i^2]$$

Standard deviation

$$\sigma_i = \sqrt{\sigma_i^2} = \sqrt{E[\varepsilon_i^2]}$$

[1] The analysis of errors presented generally assumes we are considering errors that have zero as their mean value because one assumes that otherwise constant adjustments (e.g. line loss factors) would be made to eliminate any known offset.

Derivation B (2)
Standard statistical error allocation

Covariance

Covariance, related to “ correlation” , is the average value of two zero-mean random variables multiplied together:

$$\sigma_{ij} = E[\varepsilon_i \varepsilon_j]$$

In the case when $i = j$ then this is the variance again:

$$\sigma_{ii} = \sigma_i^2$$

Derivation B (3)
Standard statistical error allocation

Correlation

Correlation is a normalised version of covariance, which gives a number between -100% and +100% indicating how closely two variables are related. For example, this is calculated by the CORREL function in Excel.

The correlation between ε_i and ε_j is ρ_{ij} which is related to covariance:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$$

This can be rearranged to express covariance in terms of correlation:

$$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$$

Derivation B (4)
Standard statistical error allocation

Problem Definition

x_i - the true value

$\tilde{x}_i$ - the measurement including some error.

ε_i - the error in the measured value, so $\tilde{x}_i = x_i + \varepsilon_i$.

ε_T - the known total of several unknown errors, such that $\varepsilon_T = \sum_j \varepsilon_j$

a_i - the error allocation fraction for $\tilde{x}_i$ in our error allocation method.

$\hat{x}_i$ - the corrected measurement of x_i, including error allocation.

Then

$$\hat{x}_i = \tilde{x}_i - a_i \varepsilon_T$$

Derivation B (5)
Standard statistical error allocation

Let δ_i be the residual error of the corrected measurement.

Then

$$\delta_i = \varepsilon_i - a_i \varepsilon_T = \varepsilon_i - a_i \sum_j \varepsilon_j$$

σ_{δ_i} is the standard deviation of the residual error.

We need to calculate the error allocation fraction a_i, that minimises σ_{δ_i}.

Derivation B (6)
Standard statistical error allocation

Calculate the variance:

$$
\begin{aligned}
\sigma_{\delta_i}^2 &= E[\delta_i^2] \\
&= E\left[\left(\varepsilon_i - a_i \sum_j \varepsilon_j\right)^2\right] \\
&= E\left[\varepsilon_i^2 - 2a_i\varepsilon_i \sum_j \varepsilon_j + a_i^2\left(\sum_j \varepsilon_j\right)^2\right] \\
&= E\left[\varepsilon_i^2 - 2a_i \sum_j \varepsilon_i\varepsilon_j + a_i^2 \sum_j \sum_k \varepsilon_j\varepsilon_k\right] \\
&= E[\varepsilon_i^2] - 2a_i \sum_j E[\varepsilon_i\varepsilon_j] + a_i^2 \sum_j \sum_k E[\varepsilon_j\varepsilon_k] \\
&= \sigma_i^2 - 2a_i \sum_j \sigma_{ij} + a_i^2 \sum_j \sum_k \sigma_{jk}
\end{aligned}
$$

Derivation B (7)
Standard statistical error allocation

Find the allocation fraction that minimises $\boldsymbol{\sigma_{\delta_i}^2}$:

Take the derivate of $\sigma_{\delta_i}^2$ with respect to a_i:

$$\frac{\partial \sigma_{\delta_i}^2}{\partial a_i} = -2\sum_j \sigma_{ij} + 2a_i \sum_j \sum_k \sigma_{jk}$$

Set the derivate to 0 and rearrange to find the minimum:

$$-2\sum_j \sigma_{ij} + 2a_i \sum_j \sum_k \sigma_{jk} = 0$$

$$a_i \sum_j \sum_k \sigma_{jk} = \sum_j \sigma_{ij}$$

$$a_i = \frac{\sum_j \sigma_{ij}}{a_i \sum_j \sum_k \sigma_{jk}}$$

Derivation B (8)
Standard statistical error allocation

Split out $\sigma_{ii} = \sigma_i^2$ to separate the variance and correlation effects:

$$a_i = \frac{\sigma_i^2 + \sum_{j\neq i} \sigma_{ij}}{\sum_j \sigma_j^2 + \sum_j \sum_{k\neq j} \sigma_{jk}}$$

Substitute in product of correlation and standard deviations to expand covariance:

$$\boxed{a_i = \frac{\sigma_i^2 + \sum_{j\neq i} \rho_{ij}\sigma_i\sigma_j}{\sum_j \sigma_j^2 + \sum_j \sum_{k\neq j} \rho_{jk}\sigma_j\sigma_k}}$$